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Epistemological Rupture in the Transition from Junior to Senior High School: An Analysis of Function Concept in Chinese Mathematics Curriculum

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Abstract

The function concept occupies a central position in school mathematics, yet its development across educational levels involves discontinuities that resist smooth cognitive progression. In the Chinese mathematics curriculum, the transition from junior to senior high school requires students to move from a "variable dependency" conception (biànliàng yīlài guān, 变量依赖观) to a "set correspondence" conception (jǐhé duìyìng guān, 集合对应观). This paper argues that such a transition constitutes what Bachelard (1938) termed an epistemological rupture, a break in which previously adequate ways of knowing become obstacles to new knowledge. Through a theoretical analysis grounded in Bachelard's epistemological obstacle, Duval's theory of semiotic representation registers, and Sfard's reification framework, this study identifies three interrelated dimensions of the rupture: ontological (the shift from process to object), semiotic (the shift from natural-language descriptions to set-theoretic formalism), and cognitive (the demand for structural conception built on inadequate operational foundations). By examining textbook definitions, curriculum standards, and the mathematical structure of the concept itself, this paper traces the specific mechanisms through which prior knowledge becomes an impediment and proposes that recognizing this rupture as structurally necessary, rather than contingently problematic, can inform more coherent instructional design at the critical juncture between school levels.

Keywords*— *function concept, epistemological rupture, semiotic representation, reification, Chinese mathematics curriculum

I. INTRODUCTION

Few mathematical ideas carry the explanatory weight of the function. From the elementary recognition that one quantity determines another, to the abstract machinery of set theory and beyond, the function concept serves as both a unifying thread and a persistent source of difficulty in school mathematics. Students encounter functions repeatedly across grade levels, yet each encounter demands not merely an extension but, in certain respects, a reconstruction of what a function is understood to be.

In China's mathematics curriculum, this reconstruction takes a particularly sharp form. During the junior high school years (grades 7 to 9), students develop a conception of function centered on the idea of *variable dependency* (变量依赖): one quantity changes as another changes. The 2024 edition of the junior high mathematics textbook published by the People's Education Press (人教版 2024, hereafter PEP-JH) defines the function in these terms: in a changing process involving two variables, if every determined value of one variable corresponds to a uniquely determined value of the other, then the second variable is called a function of the first (People's Education Press, 2024). Students at this level work with specific functions through formulas ($y = 2x$, $S = \pi r^2$), tables of values, and graphs, always anchored in concrete physical situations such as distance traveled at constant speed or the area of a circle given its radius.

When students enter senior high school (grade 10 onward), they encounter a fundamentally different definition. The senior high textbook (人教 A 版 2019, hereafter PEP-SH) states that given two non-empty sets of numbers A and B, if a definite correspondence relation f assigns to each element x in A a uniquely determined element $f(x)$ in B, then this correspondence $f: A \to B$ is called a mapping from A to B (People's Education Press, 2019). The function is now presented as a set correspondence between non-empty sets of numbers, with three essential components: the domain (定义域), the correspondence relation (对应关系), and the range (值域).

The gap between these two formulations is not merely a matter of increasing precision. The junior high conception treats functions as descriptions of covariation between measurable quantities: a car traveling at 60 km/h covers a distance $s = 60t$ as time t varies. The senior high conception treats functions as abstract mappings between sets, where the "varying" has been replaced by "belonging," and the physical process has been replaced by a formal assignment rule. A student who has spent three years understanding functions as processes of change must now understand them as objects defined by set-theoretic conditions.

This shift has been recognized in Chinese mathematics education research as a practical difficulty (Qu, 2016), yet it has rarely been subjected to systematic theoretical analysis using epistemological frameworks. Most discussions treat it as a pedagogical challenge to be managed through better bridging activities or review sessions. Such an approach, however well-intentioned, may underestimate the depth of the cognitive reorganization required.

This paper takes a different position. Drawing on Bachelard's (1938) concept of epistemological rupture, we argue that the transition from variable dependency to set correspondence is not a continuous deepening of understanding but a genuine rupture: a break in which the very knowledge that served the student well in junior high school becomes an obstacle to learning at the senior high level. To analyze this rupture, we employ three complementary theoretical lenses: Bachelard's theory of epistemological obstacles (as developed in mathematics education by Brousseau, 1983, 1997), Duval's (1995, 2006) theory of semiotic representation registers, and Sfard's (1991, 2000) theory of reification. Together, these frameworks allow us to examine the rupture at multiple levels: the epistemological structure of the obstacle, the semiotic shift in representational systems, and the cognitive transformation from process to object.

The analysis proceeds as follows. Section 2 presents the three theoretical frameworks and explains how they converge to illuminate concept transitions. Section 3 describes the specific curricular context of function teaching in China, drawing on the most recent textbook editions and curriculum standards. Section 4 constitutes the core analysis, examining the epistemological rupture through three dimensions. Section 5 discusses the implications for teaching and curriculum design, and Section 6 offers concluding remarks.

II. THEORETICAL FRAMEWORK

2.1 Epistemological Obstacle and Rupture

Gaston Bachelard (1938) introduced the concept of *epistemological obstacle* (*obstacle épistémologique*) to describe a distinctive form of resistance in the development of scientific knowledge. Unlike external difficulties (such as the complexity of a problem or the limitations of instruments), an epistemological obstacle resides within knowledge itself: it is pre-existing knowledge that constitutes the obstacle to forming new knowledge (Bachelard, 1938/2002). The obstacle is not ignorance but rather a way of knowing that was once functional and appropriate but has become inadequate in a new context.

Bachelard identified several types of epistemological obstacles: the obstacle of first experience (generalization from limited instances), the verbal obstacle (confusion arising from language), the substantialist obstacle (the tendency to reify processes into substances), the pragmatic obstacle (the preference for what is useful over what is true), and the animist obstacle (the projection of life and intention onto inert matter). These obstacles are not errors to be eliminated but structural features of knowledge development. For Bachelard, knowing requires

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rectifying one's knowledge (*connaître, c'est rectifier une connaissance*) (Bachelard, 1938/2002).

Guy Brousseau (1983, 1997) adapted Bachelard's concept to mathematics education, arguing that obstacles to learning mathematics are not merely the result of teaching deficiencies but arise from the very nature of mathematical knowledge. Brousseau distinguished between ontological obstacles (related to the nature of mathematical objects), instrumental obstacles (related to the means of approaching those objects), and epistemological obstacles (related to the ways of knowing the objects themselves). The key insight is that obstacles are inherent in the acquisition of knowledge and constitutive of knowledge itself (Brousseau, 1997). In this view, encountering and overcoming obstacles is not a detour from learning but an essential part of it.

Sierpinska (1992) applied this framework to the understanding of the function concept, demonstrating that students' difficulties are not arbitrary but follow predictable patterns rooted in the historical development of the concept and in the inherent tensions within the concept itself. She showed that the evolution from the Leibnizian notion of function (as a quantity associated with a curve) through the Eulerian notion (as an analytic expression) to the Dirichlet-Bourbaki notion (as a correspondence between sets) involved genuine epistemological ruptures at the level of the discipline, and that analogous ruptures appear in individual learning.

The concept of *epistemological rupture* (*rupture épistémologique*) is closely related to that of epistemological obstacle. Whereas the obstacle names the resistance encountered, the rupture names the break that must occur for knowledge to progress. In Bachelard's philosophy, the rupture is the moment of overcoming: the point at which prior knowledge is not discarded but fundamentally restructured. Bachelard insisted that all knowledge is a rectification of prior knowledge (Bachelard, 1938/2002), meaning that intellectual progress requires a negation of earlier modes of thought that were themselves necessary at an earlier stage. It is precisely this dual character, of being both necessary and insufficient, that makes the transition from junior to senior high school function concept a paradigmatic case of epistemological rupture.

2.2 Semiotic Representation Registers

Raymond Duval (1995, 2006) developed a theory of semiotic representation that provides tools for analyzing how mathematical objects are accessed through different sign systems. The theory rests on a fundamental thesis: mathematical objects are never given directly but only through their representations (Duval, 2006). This means that understanding a mathematical concept requires the ability to move among different representational systems, or *registers*, such as natural language, algebraic notation, graphical displays, and tabular forms.

Duval distinguished three cognitive activities associated with representations: *formation* (producing a representation within a given register), *treatment* (transforming a representation within the same register, such as simplifying an algebraic expression), and *conversion* (transforming a representation from one register to another, such as converting an algebraic formula into a graph). Of these, conversion is the most cognitively demanding and the most essential for mathematical understanding. A student who can perform treatments within a single register but cannot convert between registers possesses only a partial understanding of the concept (Duval, 1995).

Duval also introduced the concept of *noesis*, the act of apprehending or conceptualizing a mathematical object through its representations, and distinguished it from the *semiosis*, the act of producing or manipulating representations. Understanding requires the coordination of noesis and semiosis: the student must recognize that different representations refer to the same underlying mathematical object. When this coordination fails, what Duval called a *rupture* occurs: the student treats different representations as referring to different objects, or treats a single representation as the object itself.

The notion of *congruence* is also relevant. Two representations in different registers are *congruent* when there is a one-to-one correspondence between their constituent elements. When congruence is low, conversion becomes more difficult. For instance, converting the algebraic expression $y = 2x + 3$ to a graph is relatively straightforward because the elements of the formula correspond clearly to visual features of the line. Converting a verbal description to a graph is less congruent because the relationship between the linguistic elements and the graphical features is not one-to-one.

2.3 Reification: From Process to Object

Anna Sfard (1991, 2000) developed a theory of mathematical concept formation that describes how learners progress from operational (process-based) to

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structural (object-based) conceptions. Sfard argued that many mathematical concepts have a dual nature: they can be understood either as processes (dynamic operations) or as objects (static entities). The function concept is perhaps the paradigmatic example: one can think of $f(x) = 2x + 3$ as a procedure for computing an output from an input, or as an object, a function that can be manipulated, compared, and composed.

Sfard proposed three stages in the development of a mathematical concept:

Interiorization. The learner becomes familiar with a concept at the operational level, performing processes smoothly and efficiently. At this stage, the concept exists only as a set of actions or procedures. For the function concept, interiorization means being able to evaluate $f(x)$ for specific values, plot points on a graph, or trace the behavior of a formula.

Condensation. The learner begins to perceive the process as a whole, without needing to carry out each step. The process is "compressed" into a mental unit that can be manipulated without being executed step by step. At this stage, the student can think about the function $f(x) = 2x + 3$ without computing individual values, perhaps recognizing that it represents a linear relationship or that its graph is a straight line.

Reification. The learner suddenly becomes aware of the process as a manipulable object. This is the most difficult and most important stage, characterized by what Sfard called an "ontological shift." The concept is no longer something the student *does* but something the student *has*. The function becomes an object that can be an element of a set, a member of a function space, or an argument of another function (such as the derivative operator).

Sfard (1991, 2000) emphasized that reification is not a gradual process but a sudden restructuring: the concept is born when the process is suddenly seen as an object (Sfard, 1991). The difficulty of reification lies precisely in the fact that the operational conception, which was perfectly adequate at the earlier stage, now becomes an obstacle. The student who thinks of a function only as a computational procedure cannot easily grasp that the function itself can be treated as a single mathematical entity.

Sfard and Linchevski (1994) demonstrated this dynamic in the context of algebra, showing how students' operational conceptions of algebraic expressions blocked their ability to treat expressions as objects. Their analysis is directly applicable to the function concept, where a similar blockage occurs when students must move from evaluating functions to manipulating functions as objects.

2.4 Convergence of the Three Frameworks

The three frameworks converge in their treatment of concept transitions. Bachelard's epistemological obstacle describes the *epistemological structure* of the difficulty: prior knowledge becomes an impediment. Duval's theory describes the *semiotic dimension*: the rupture involves a shift in representational systems that requires new cognitive operations. Sfard's theory describes the *cognitive mechanism*: the transition from process to object requires a reorganization of the learner's conception.

For the transition from junior high to senior high school function concept, these three perspectives yield a comprehensive analytical framework. The epistemological lens reveals why students' prior knowledge of variable dependency becomes an obstacle to set correspondence. The semiotic lens reveals the representational discontinuity between the registers used in the two school levels. The cognitive lens reveals the ontological shift from process to object that the transition demands. Together, they allow us to analyze the rupture not as a contingent pedagogical problem but as a structurally necessary break inherent in the mathematical nature of the function concept itself.

III. THE FUNCTION CONCEPT IN THE CHINESE MATHEMATICS CURRICULUM

3.1 Junior High School: The Variable Dependency Conception

In the Chinese mathematics curriculum, the function concept is first formally introduced in grade 8 (junior high year 2), within the "Number and Algebra" domain. The 2022 edition of the Compulsory Education Mathematics Curriculum Standards specifies that students should explore the relations and rules among quantities in concrete situations, understand the concept of function in real-world contexts, and be able to express functions using formulas, tables, and graphs (Ministry of Education, 2022).

The PEP-JH textbook (2024 edition, Volume 2, Chapter 22) presents the function concept through a sequence of concrete situations. The first situation involves a car traveling at constant speed, where the distance s is related to the time t by the formula $s = 60t$ (People's Education Press, 2024). Subsequent situations include the relationship between the side

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length and area of a square ($S = x^2$), the relationship between water height and time in a filling tank, and others (People's Education Press, 2024). After presenting several such situations, the textbook abstracts a common pattern: in each situation, there are two variables, and when one variable takes a determined value, the other is uniquely determined. This relationship is called a function (函数). More precisely, in a changing process with two variables x and y, if every determined value of x corresponds to a uniquely determined value of y, then x is the independent variable (自变量) and y is a function of x (因变量) (People's Education Press, 2024).

This definition emphasizes several features. First, the function is understood through *process*: it exists "in a changing process" (在一个变化过程中). Second, the emphasis is on *dependence*: one quantity varies as another varies. Third, the uniqueness condition ("uniquely determined value") is stated, but it is embedded in a dynamic, quasi-physical context. The student's understanding at this level can be characterized as *variable dependency*: y is a function of x means that y depends on x in a specific, deterministic way.

During grades 8 and 9, students study three families of specific functions: linear functions ($y = kx + b$, $k \neq 0$), inverse proportional functions ($y = k/x$, $k \neq 0$), and quadratic functions ($y = ax^2 + bx + c$, $a \neq 0$) (Ministry of Education, 2022; People's Education Press, 2024). For each family, they learn to derive properties from formulas, sketch graphs, and apply functions to solve problems. Throughout, the emphasis remains on the dynamic relationship between variables: students trace how y changes as x changes, observe the resulting patterns in graphs, and solve equations and inequalities by reading values from these graphs.

The cognitive achievement at the end of junior high school is substantial. Students can evaluate functions, plot and interpret graphs, determine domains from contexts, and solve problems using function properties. Yet this achievement rests entirely on the variable dependency conception. The function is a process of covariation, anchored in physical situations, expressed through formulas and graphs, and understood primarily through its dynamic behavior. As Tall and Vinner (1981) observed, a learner's *concept image*, the total cognitive structure associated with a concept, may diverge significantly from the formal concept definition. At the junior high level, the concept image of function is built from processes and physical contexts, creating a structure that will prove difficult to reconcile with the set-theoretic definition encountered later.

3.2 Senior High School: The Set Correspondence Conception

When students enter senior high school (grade 10), the function concept undergoes a fundamental redefinition. The PEP-SH textbook (2019 edition, Compulsory Volume 1, Chapter 3) begins by revisiting the junior high conception only to supersede it:

The textbook notes that the junior high conception of function, studied from the perspective of variable dependency, captures the essential feature of functions but is limited in scope, and that the language of sets allows a more general and rigorous definition (People's Education Press, 2019).

The new definition is then stated:

The definition states that given two non-empty sets of numbers A and B, if a definite correspondence relation f assigns to each element x in A a uniquely determined element $f(x)$ in B, then $f: A \to B$ is called a function from A to B, written $y = f(x)$, $x \in A$ (People's Education Press, 2019).

This formulation differs from the junior high definition in several respects that bear careful analysis.

First, the ontological status of the function has changed. Instead of a relationship between *variables* that vary in a process, the function is now a correspondence between *sets* of numbers. The variables x and y are no longer quantities that change but elements that *belong* to sets. The phrase "in a changing process" has disappeared; in its place stands "let A and B be two non-empty sets of numbers." The function is no longer something that happens but something that exists, a mathematical object characterized by three components: the domain A, the correspondence relation f, and the range $\{f(x) \mid x \in A\} \subseteq B$.

Second, the language has shifted from the informal register of variable dependency to the formal register of set theory. Terms such as "set" (集合), "element" (元素), "correspondence" (对应), and "mapping" (映射) replace the language of "changing process," "variable," and "dependence." This represents not merely a change of vocabulary but also a change in the representational system: the mathematical objects are now conceived within a different theoretical framework.

Third, the scope of the definition has expanded. The junior high conception, tied to physical processes, naturally encompasses only functions that can be expressed by formulas relating measurable quantities. The set-theoretic conception, by contrast, allows functions that have no simple formula: for instance, the Dirichlet function (which equals 1 for rational inputs and 0 for irrational inputs), piecewise-defined functions with complex structures, and functions defined by tables without any discernible pattern (People's Education Press, 2019).

During the first year of senior high school, students study function properties (monotonicity, odd/even symmetry, maximum/minimum values) within this set-theoretic framework (People's Education Press, 2019). They learn to determine domains rigorously using inequalities on sets, to compose and invert functions using the correspondence structure, and to prove properties using the formal definition. The emphasis shifts from *tracing how quantities change* to *analyzing the structural properties of a mathematical object*.

3.3 The Curricular Gap

The two curriculum documents that govern function teaching at each level make the discontinuity explicit. The Compulsory Education Mathematics Curriculum Standards (2022 edition) requires students to explore simple numerical changes and functional relationships in concrete situations (Ministry of Education, 2022), emphasizing intuitive understanding and practical application. The Senior High School Mathematics Curriculum Standards (2017 edition, revised 2020) requires students to establish the function concept using set theory and correspondence language, understand function properties, and use function concepts and methods to solve problems (Ministry of Education, 2020), emphasizing formal definition and abstract reasoning.

Between these two documents lies a content gap (内容断层): the junior high curriculum cultivates a dynamic, contextually grounded understanding, while the senior high curriculum demands a static, formally grounded understanding. The gap is not a matter of degree (more depth, more complexity) but of kind: the very conception of what a function *is* must be restructured.

This is the rupture we analyze in the following section.

IV. ANALYSIS OF THE EPISTEMOLOGICAL RUPTURE

4.1 The Ontological Rupture: From Process to Object

The most fundamental dimension of the rupture is ontological. At the junior high level, the function is a *process*: a description of how one quantity varies with another. The textbook's language makes this clear: "In a changing process" (在一个变化过程中), y depends on x. The function is not an object that exists independently of this process; it is the process itself, or rather, the regularity governing the process. When a student says "$y = 2x + 3$ is a function," what is meant is that a certain process of doubling and adding is occurring.

At the senior high level, the function is an *object*: a correspondence f: $A \to B$ defined by three components. It is no longer something that *happens* but something that *is*. The student is expected to treat the function as a mathematical entity that can be an element of a set (for instance, the set of all continuous functions on [0, 1]), an argument of an operation (for instance, the derivative operator applied to f), or the subject of a proposition ("f is continuous," "f is increasing").

This shift maps directly onto Sfard's (1991) framework. The junior high student has achieved *interiorization*: they can perform the processes of evaluating functions, plotting points, and reading values from graphs. Some students have reached *condensation*: they can perceive a function as a whole (for instance, recognizing that $y = 2x + 3$ represents a straight line with slope 2 without computing individual points). But the senior high curriculum demands *reification*: the function must become a structural object that can be manipulated independently of the process it encodes.

The difficulty of this reification can be analyzed more precisely using Sfard's observation that the reified concept is not a negation of the operational one but subsumes it (Sfard, 1991). The student must not abandon their process conception but must recognize it as one aspect of a more general object conception. The problem is that the junior high curriculum provides almost no opportunity for this recognition. Students work exclusively with functions as processes of covariation, in physical contexts, expressed through formulas and graphs. They never encounter a function as an object in its own right.

Consider a concrete illustration, drawn from the respective textbook exercises (People's Education Press, 2024; People's Education Press, 2019). At the junior high level, the question "Is y a function of x?" is answered by checking whether each value of x yields a unique value of y in a given formula or table. At the senior high level, the same question requires checking whether a given *correspondence* satisfies the condition that every element of set A is mapped to a unique element of set B. The difference is not merely one of formality. In the first case, the student is verifying a process: does this formula produce a unique output for each input? In the second case, the student is verifying a property of an object: does this correspondence satisfy the defining condition of a function? The cognitive operation is fundamentally different.

This ontological rupture generates what Bachelard would call a *substantialist obstacle*. The student, accustomed to thinking of a function as a process, encounters the demand to treat it as a substance, an entity with fixed properties. This is analogous to the historical rupture in mathematics itself: it took centuries for mathematicians to move from the Newtonian conception of function (as a quantity varying in time) through the Eulerian conception (as an analytic expression) to the Dirichlet-Bourbaki conception (as a set-theoretic correspondence). At each historical stage, the previous conception served as an obstacle to the next. The student recapitulates this history, and the obstacles are structurally similar.

The ontological rupture can also be described in terms of Tall's (1991) concept of *procept*: a mathematical entity that is simultaneously a process and a concept (object). The function $f(x) = 2x + 3$ is a procept: it denotes both the process of "doubling and adding three" and the object "the function with formula $2x + 3$." At the junior high level, the process pole dominates; the student thinks of the function as something to *do*. At the senior high level, the object pole must dominate; the student must think of the function as something to *think about*. The difficulty lies precisely in the fact that the proceptual nature of the function, which was barely visible at the junior high level, must now become the central focus of attention.

4.2 The Semiotic Rupture: From Natural Language to Set-Theoretic Formalism

The ontological shift is accompanied by, and in some sense constituted by, a semiotic rupture. Duval's (1995, 2006) framework allows us to describe this rupture with precision.

At the junior high level, the function concept is expressed through four main semiotic registers:

1. **Natural language**: descriptions such as the distance s increasing as the time t increases, or y changing as x changes.
2. **Algebraic formulas**: $s = 60t$, $y = 2x + 3$, $S = \pi r^2$.
3. **Graphical representations**: Plots of specific functions on coordinate axes.
4. **Numerical tables**: Tables of corresponding x and y values.

Throughout the junior high curriculum, the natural-language register plays a dominant role. The textbook introduces the function concept through verbal descriptions of physical situations, and the definition itself uses natural language, stating that for every determined value of x, there is a uniquely determined value of y. The formulas are always linked to specific situations, and the graphs are always interpreted as descriptions of physical processes. Conversions between registers are guided by the physical context: the student converts a word problem to a formula, evaluates the formula to produce a table, and plots the table to produce a graph, always anchored in the original situation.

At the senior high level, a new register appears and quickly becomes dominant: the **set-theoretic formal language**. The function is now written $f: A \to B$, with explicit reference to domain A, codomain B, and correspondence rule f. Properties are stated using set notation, such as specifying the domain as the set $\{x \mid x > 0\}$ or stating that the function is increasing on the interval $(0, +\infty)$ (People's Education Press, 2019). The correspondence relation is described not as a formula but as an abstract mapping: $x \mapsto f(x)$.

The discontinuity between these two semiotic regimes can be analyzed using Duval's concept of *congruence*. At the junior high level, the congruence between registers is relatively high. The formula $s = 60t$ corresponds directly to the verbal description "distance equals speed times time," and the graph of this formula corresponds directly to the picture of a car traveling at constant speed. The elements of each register map transparently onto the physical situation that grounds them all.

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At the senior high level, the congruence drops sharply. The set-theoretic notation f: $A \rightarrow B$ has no direct counterpart in natural language. The expression "correspondence relation" (对应关系) is not equivalent to "rule" or "formula"; it is a new term that refers to an abstract structural relationship between sets. The domain A is not simply "the values of x" but a set defined by membership conditions. Converting between the set-theoretic register and the algebraic register, or between the set-theoretic register and the graphical register, requires operations that are *non-congruent*: there is no one-to-one correspondence between the elements of the notation f: $A \rightarrow B$ and the elements of a graph or a formula.

This non-congruence has a concrete consequence. At the junior high level, conversions between registers are *transparent*: the student can see that the formula, the graph, and the verbal description all refer to the same situation because they are all grounded in the physical context. At the senior high level, conversions become *opaque*: the student must recognize that f: $\{1, 2, 3\} \rightarrow \{4, 6, 8\}$ with $f(x) = 2x$, the set of ordered pairs $\{(1, 4), (2, 6), (3, 8)\}$, and the arrow diagram with arrows from 1 to 4, 2 to 6, and 3 to 8 all represent the same function (People's Education Press, 2019). The common referent is no longer a physical situation but an abstract mathematical object, and the student must coordinate multiple representations of this object without the support of physical intuition.

Duval (1995) argued that the ability to convert between non-congruent representations is the hallmark of genuine mathematical understanding. When conversion fails, the student may possess representations in each register separately but lacks the coordination that constitutes the concept. This is precisely the situation that arises at the junior-to-senior-high transition. Students arrive with well-developed abilities to work with functions in the natural-language, algebraic, graphical, and tabular registers. But when the set-theoretic register is introduced, their existing conversion abilities are inadequate. They may be able to read a graph or manipulate a formula, but they cannot convert between these representations and the set-theoretic notation. The result is what Duval would call a *rupture in semiosis*: the semiotic activity at the senior high level is disconnected from the semiotic activity at the junior high level, and the noetic activity (the apprehension of the mathematical object) is correspondingly disrupted.

4.3 The Cognitive Rupture: The Variable Dependency Conception as Epistemological Obstacle

The ontological and semiotic ruptures described above converge at the cognitive level. The variable dependency conception, which was a genuine cognitive achievement at the junior high level, becomes an *epistemological obstacle* at the senior high level. We can identify several specific mechanisms through which this occurs.

Obstacle 1: The process conception blocks the object conception.

The junior high student understands a function as a process of covariation: y changes as x changes. This conception supports a wide range of mathematical activity, from evaluating formulas to interpreting graphs. But when the senior high curriculum requires the student to treat a function as an object (for instance, to determine whether two functions are identical by comparing their domains and correspondence relations), the process conception becomes an impediment. The student who thinks of $f(x) = x^2$ as "squaring the input" cannot easily see that $f = g$ if and only if f and g have the same domain and the same correspondence rule, because the process conception does not provide the categories "domain" and "correspondence rule" as components of the function-object.

A typical manifestation, found in both textbooks and standardized assessments (People's Education Press, 2019): a student is asked whether $f(x) = x$ and $g(x) = x^2/x$ are the same function. Using the process conception, both functions "give back x," so they should be the same. But using the set-theoretic conception, they have different domains (g is undefined at 0), so they are different functions. The variable dependency conception, which attends only to what the function *does*, cannot resolve this question because it has no category for the domain as a component of the function-object.

Obstacle 2: The context-dependence blocks abstraction.

At the junior high level, every function is introduced through a physical context: distance and time, area and radius, cost and quantity. This context provides meaning and motivation, but it also constrains the student's conception. The function is *about* something: it describes a relationship in the physical world. The idea of a function that has no physical

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referent, or whose domain is an arbitrary set of numbers, is alien to the junior high conception.

When the senior high textbook presents a function such as f: {1, 2, 3, 4, 5} → {2, 4, 6, 8, 10} with $f(x) = 2x$ (People's Education Press, 2019), the student seeks a physical situation that this function describes. If none is forthcoming, the student experiences confusion. The set-theoretic conception requires the student to accept that a function is defined purely by its domain, its correspondence rule, and its range, without reference to any physical situation. This abstraction represents what Bachelard (1938/2002) called a *rupture with the concrete*: the break from immediate experience that is necessary for scientific thought.

Obstacle 3: The uniqueness criterion is reinterpreted.

Both the junior high and senior high definitions include a uniqueness condition: for every value of x, there is a unique value of y. But the role of this condition differs between the two levels. At the junior high level, uniqueness is verified through the physical context: given a time t, there is only one distance s at that instant. The criterion is grounded in physical determinism. At the senior high level, uniqueness is a formal condition on the correspondence: each element of A is mapped to exactly one element of B. The criterion is now a structural property of the mapping, independent of any physical interpretation.

Students often carry the junior high interpretation into the senior high context, attempting to verify uniqueness by imagining a physical process rather than analyzing the formal structure of the correspondence. This generates errors when the function has no natural physical interpretation, or when the domain and range are abstract sets of numbers.

Obstacle 4: The graphical interpretation is disrupted.

At the junior high level, the graph of a function is understood as a picture of the process: the curve *rises* because y is *increasing*, it *falls* because y is *decreasing*. The graphical representation is read as a narrative of change. At the senior high level, the graph must be understood as a set of ordered pairs $\{(x, f(x)) \mid x \in A\}$, and properties such as monotonicity must be defined using set-theoretic language (People's Education Press, 2019): f is increasing on interval I if for any $x_1, x_2 \in I$ with $x_1 < x_2$, we have $f(x_1) < f(x_2)$.

The senior high definition of monotonicity is not simply a more precise version of the junior high understanding. It represents a fundamentally different way of seeing the graph: not as a picture of motion but as a set of points satisfying a universal condition. The student who reads monotonicity off the graph visually ("the curve goes up") must learn to prove it using the formal definition, which requires a different relationship to the graphical register entirely. The graph is no longer an illustration of a process but a representation of a set-theoretic structure.

Obstacle 5: The variable itself is reconceptualized.

In the junior high conception, variables are quantities that vary: x is a number that changes, and y changes in response. At the senior high level, x is an element of a set: a fixed but arbitrary member of the domain. The shift from "variable" (*biànliàng*, literally "changing quantity") to "element of a set" (*shùjí de yuánsù*) is subtle but fundamental. The variable is no longer something that *moves* through a range of values but something that *ranges over* a set. This is an instance of what Bachelard called the transition from the "substantialist" to the "formal" mode of thought: the variable is no longer a substance with dynamic properties but a placeholder in a formal structure.

The term "variable" (*biànliàng*) itself becomes an obstacle. Its Chinese name, meaning "changing quantity," carries the implication of process and motion. When the senior high textbook asks the student to treat x as an element of a set, the student must suppress the dynamic connotations of the term and replace them with a static, set-theoretic understanding. This suppression is not a matter of simple substitution; the old meaning persists as an undercurrent, distorting the student's engagement with the new formalism.

4.4 The Rupture as Structurally Necessary

The analysis above reveals that the transition from variable dependency to set correspondence is not a smooth deepening of understanding but a genuine rupture involving multiple dimensions. This raises a question: is this rupture a contingent feature of the Chinese curriculum, or does it reflect a deeper structural necessity?

We argue for the latter. The function concept, as it has developed in the history of mathematics, necessarily involves these ruptures. Bachelard (1938/2002) argued that the development of scientific concepts follows a pattern of rupture and rectification:

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each stage of knowledge contains the seeds of its own overcoming. The function concept illustrates this pattern. The Leibnizian conception (function as a quantity associated with a curve) was overcome by the Eulerian conception (function as an analytic expression), which was in turn overcome by the Dirichlet-Bourbaki conception (function as a set-theoretic correspondence). Each rupture was not an arbitrary change but a rectification of the preceding knowledge, motivated by internal demands of mathematical development.

The student's cognitive development recapitulates this historical sequence, not by coincidence but because the logical structure of the concept demands the same sequence of ruptures. The variable dependency conception is the student's "Leibnizian" stage: functions are understood through their manifestations in specific curves and processes. The set correspondence conception is the "Dirichlet-Bourbaki" stage: functions are understood as abstract correspondences between sets. Between these stages lies a rupture that is not optional but structurally required by the mathematical content itself.

This does not mean that the rupture is insurmountable or that curriculum design is irrelevant. On the contrary, recognizing the rupture as structurally necessary allows educators to design instructional interventions that address its specific dimensions. If the rupture is merely a contingent gap, it can be bridged by review or acceleration. If it is a structural necessity, it requires a pedagogical approach that acknowledges the cognitive reorganization and provides appropriate support for the ontological shift.

V. DISCUSSION

5.1 Implications for Teaching

The analysis presented in Section 4 carries several implications for the teaching of the function concept at the junior-to-senior-high transition.

First, the ontological dimension of the rupture suggests that simply presenting the set-theoretic definition and moving on is insufficient. The student must undergo a genuine cognitive restructuring, not merely acquire new information. Sfard (1991) noted that reification is typically preceded by a period of difficulty and confusion during which the learner struggles to reconcile the new structural conception with the old operational one. Teachers should anticipate this period and provide tasks that explicitly require the structural conception. For instance, tasks that ask students to compare functions not by their formulas but by their domains and correspondence relations force the student to attend to the function as an object rather than as a process.

Second, the semiotic dimension of the rupture suggests that the transition between representational systems requires explicit instructional attention. Duval (1995) argued that conversion between non-congruent representations is the most demanding cognitive activity in mathematics learning. At the transition point, students must learn to convert between the natural-language and algebraic registers of junior high school and the set-theoretic register of senior high school. Teachers can support this by providing tasks that require such conversions explicitly: for instance, translating between the formal statement that for every x in A there exists a unique $f(x)$ in B and the arrow diagram representing the same correspondence, or between the set-builder notation for the domain and the verbal description of the conditions on x.

Third, the epistemological dimension of the rupture suggests that teachers should acknowledge the difficulty explicitly rather than treating it as a matter of students "not trying hard enough" or "not reviewing enough." Brousseau (1997) emphasized that epistemological obstacles are constitutive of knowledge itself, and that the teacher's role is not to remove obstacles but to create conditions under which students can overcome them. In the context of the function concept, this means creating tasks that force students to confront the inadequacy of their variable dependency conception. A well-designed task might present a correspondence that satisfies the set-theoretic definition of function but has no obvious physical interpretation (such as a function defined by a lookup table with no discernible pattern) and ask students to determine whether it is a function. Such a task forces the student to abandon the search for physical meaning and to engage with the formal definition directly.

5.2 Implications for Curriculum Design

The analysis also has implications for curriculum design at both the junior high and senior high levels.

At the junior high level, the variable dependency conception could be enriched with elements that anticipate the set-theoretic conception without requiring full reification. For instance, teachers could introduce the idea that a function has a *domain*

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(the set of allowable inputs) and a *range* (the set of possible outputs) even within the variable dependency framework. The 2024 PEP-JH textbook already mentions domain in the context of physical constraints (for instance, the radius of a circle must be positive; People's Education Press, 2024), but this notion could be developed more systematically as a component of the function concept rather than merely a contextual constraint. Similarly, the idea of a *correspondence* could be introduced through simple mapping diagrams (arrow diagrams) that show how elements of one set correspond to elements of another, without yet using set-theoretic language.

At the senior high level, the curriculum could acknowledge the rupture explicitly and provide a transitional period in which the variable dependency and set correspondence conceptions coexist. Rather than presenting the set-theoretic definition as a replacement for the junior high conception, the curriculum could present it as a *generalization* that encompasses the junior high conception as a special case. The historical development of the function concept provides a natural narrative for this: Leibniz → Euler → Dirichlet → Bourbaki, each stage generalizing the preceding one. By situating the student's own transition within this historical sequence, the curriculum can help the student understand that the rupture is not a failure of their junior high learning but a natural step in the development of mathematical knowledge.

5.3 Implications for Research

This paper has offered a theoretical analysis of the epistemological rupture in the function concept transition. The analysis generates several questions for future empirical research:

1. How do students actually work through the transition from variable dependency to set correspondence? Do they exhibit the patterns of obstacle and rupture predicted by the theoretical framework?

2. What specific instructional interventions are most effective in supporting students through the rupture? Can task sequences designed with the three dimensions of the rupture in mind improve student outcomes? Recent methodological advances in tracing learning through combined process-object theories and individual learning trajectories (Bednorz et al., 2026) offer promising tools for such empirical investigations.

3. How does the rupture manifest in different curriculum systems? The Chinese curriculum provides a particularly sharp case, but analogous transitions exist in other educational systems (for instance, the transition from covariation to correspondence approaches in many Western curricula).

4. How do teachers' own conceptions of function influence their treatment of the transition? Research on teachers' analogies for function (Hatisaru, 2024) suggests that teachers' conceptions may privilege either the correspondence or the covariation perspective, and this may affect how they mediate the transition for students.

5.4 Limitations

This study is a theoretical analysis, not an empirical investigation. The claims about student cognition are based on theoretical reasoning and on the analysis of textbooks and curriculum standards, not on direct observation of student learning. The identification of specific obstacles is derived from the theoretical frameworks rather than from systematic data collection. Future empirical research is needed to test and refine the theoretical claims made here.

Additionally, this analysis focuses on the function concept as defined in two specific textbook series (PEP-JH 2024 and PEP-SH 2019). Other textbook series may present the transition differently, and the analysis may not generalize to all Chinese mathematics classrooms. The role of teachers as mediators of the curriculum is also not addressed, although it is likely to be significant in practice.

VI. CONCLUSION

The transition from junior high to senior high school in the Chinese mathematics curriculum requires students to reconstruct their understanding of the function concept from a variable dependency conception to a set correspondence conception. This paper has argued, using Bachelard's concept of epistemological rupture together with Duval's theory of semiotic representation and Sfard's theory of reification, that this transition is not a smooth progression but a genuine rupture involving three interrelated dimensions:

The *ontological* dimension: the function must be reconceived from a process of covariation to an object defined by set-theoretic conditions. This shift

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requires what Sfard (1991) called reification, the sudden restructuring of a process into an object.

The *semiotic* dimension: the student must learn to work in a new representational system (set-theoretic formalism) that is not congruent with the registers used at the junior high level (natural language, formulas, graphs, tables). The non-congruence between the old and new registers creates additional cognitive demands that go beyond the ontological shift alone.

The *cognitive* dimension: the variable dependency conception, which was a genuine achievement at the junior high level, becomes an epistemological obstacle at the senior high level. Specific mechanisms of this obstacle include: the process conception blocking the object conception, context-dependence blocking abstraction, the reinterpretation of the uniqueness criterion, the disruption of graphical interpretation, and the reconceptualization of the variable itself.

These three dimensions are not independent. The ontological shift is made more difficult by the semiotic rupture (the new language makes the new conception harder to access), and both are intensified by the cognitive obstacle (the old conception persists and distorts engagement with the new material). Together, they constitute a rupture that is structurally necessary given the mathematical nature of the function concept itself.

The practical implication is clear: the transition cannot be managed by simply presenting the new definition and expecting students to adapt. It requires deliberate instructional design that addresses each dimension of the rupture. Teachers and curriculum designers should anticipate the obstacle, support the semiotic conversion, and create conditions for the ontological shift to occur. The historical development of the function concept, from Leibniz through Dirichlet to Bourbaki, provides both a justification for the difficulty and a narrative that can help students understand their own cognitive journey as part of the broader development of mathematical thought.

Bachelard (1938/2002) observed that science unquestionably progresses, but it does not progress continuously. The development of the function concept in school mathematics, we have argued, follows this same pattern of discontinuous progress. Recognizing the rupture as a feature of mathematical cognition, rather than a bug, is the first step toward designing more effective instruction at this critical juncture.

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